On Crypto-Backed Loans

Frontiers in Decentralized Finance - May 26th 2023, Zhaw School of Management and Law

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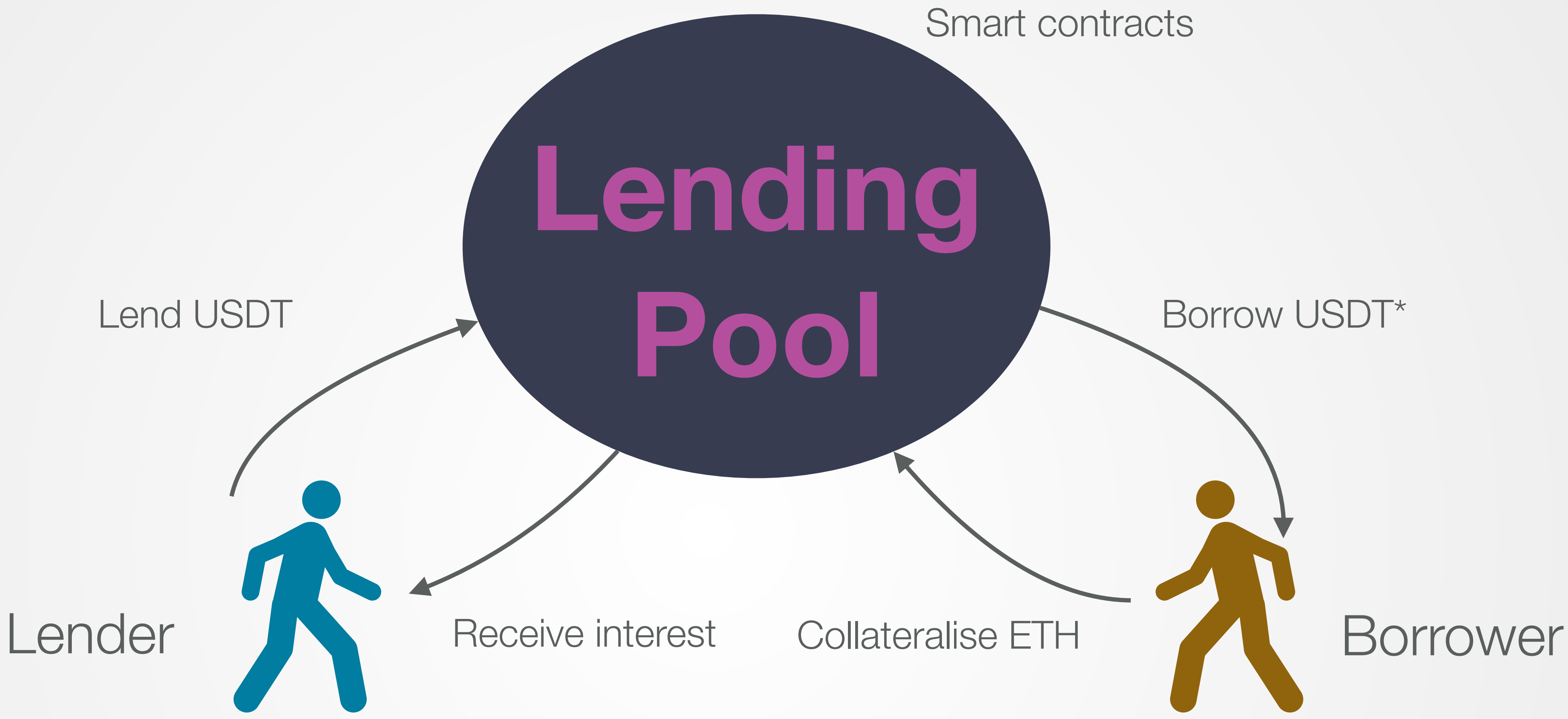
Crypto-Backed Lending



- Without intermediaries like banks
- Total value locked (on-chain platforms) as of 26th April 2023 (source):
  - AAVE $5.39b
  - JustLend DAO $3.86b
  - Compound $1.95b
- Total Value Locked: Value of digital assets on a protocol

Market landscape

On-chain

- AAVE
- JustLend.
- Compound

Off-chain

- nexo
- Unchained
- YOUHODLER

On-chain AAVE

Borrower

Your supplies Hide —

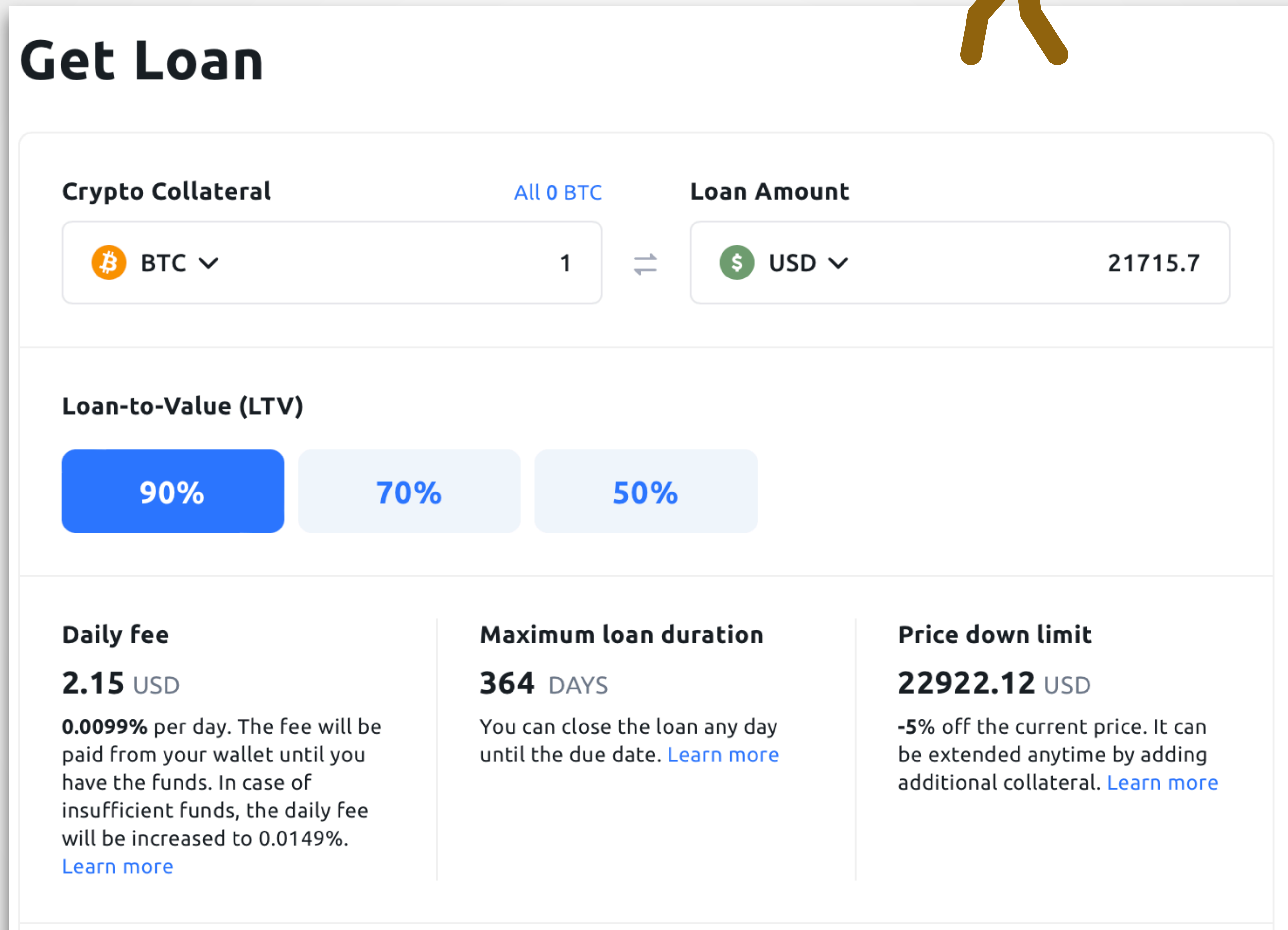
Balance $ 2,089.78 | APY 1.63 % | Collateral $ 2,089.78

Asset	Balance	APY	Collateral	
ETH	1.07 $ 2,089.78	1.63 %	(on)	Withdraw / Swap

Asset	Available	APY, variable	APY, stable	
USDT	1,732.17	3.58 %	12.48 %	Borrow / Details
DAI	1,731.55	3.59 %	12.49 %	Borrow / Details

Off-chain YouHodler

Borrower



Common features

- Non-recourse loan
- Over-collateralisation
  - >5% haircut
  - Enforced throughout the loan period
  - Maintained by liquidation/margin call with penalty (~5%) to borrower
- Borrowing rate is driven by demand and supply ≻ improve

Borrowers' motivations…?

Not easy to justify borrowers' motivation

- Loan terms seem *harsh*
  - over-collateralisation
  - liquidation and its penalty
  - interest rate payment
- Cryptos are *not* consumable/spendable
- Require management efforts

Borrowers' can

- Receive liquidity in *preferred* cryptos
- Avoid tax events and price slippage
- Hedge/speculate ≻ Derive a borrowing rate from this angle

Outline

1. Motivation ✓
2. Literature review
3. Mechanisms
4. Borrower's payoff
5. Fair borrowing premium
6. Numerical results

Non-recourse loans

Stock loans ≻ Financial engineering perspective

Xia & Zhou (2007, Mathematical Finance)

- Compute a "fair value" $f(S_0)$ as *perpetual call option*
- Denote $\tilde{S}_t = e^{-rt}S_t$ to tackle the exponentially growing barrier and strike
- Explain the difference between fair value and initial portfolio with service fee charged by lender, i.e. $f(S_0) = S - K + c$

Other stock loans literatures:

- Siu, Yam & Zhou (2014) compare effects by margin requirement and right of calling loans
- Lu & Putri (2016) include margin and finite maturity features
- Wong & Wong (2014) include stochastic volatility
- Cai & Sun (2014) include jumps
- Dai & Xu (2011) suggest an optimal redeeming strategy
- Liang et al (2010) study the effect of upper barrier

Non-recourse loans

Repurchase agreement (Repo) ≻ Explain “specialness”

Duffie (1997, The Journal of Finance)

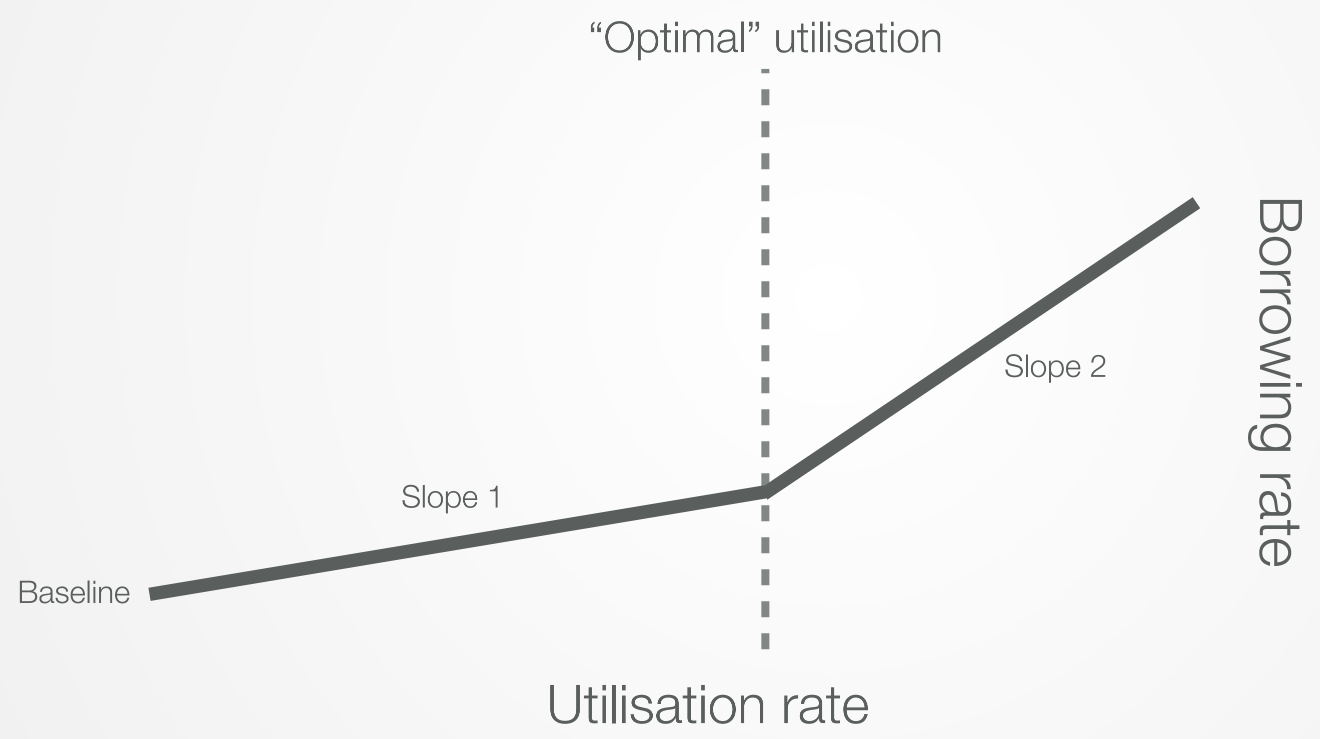
- Specialness: Repo rate being significantly below the risk free rate
- Attributes specialness to institutional factors
- Arbitrage pricing theories apply for special Repo rate

Other literatures:

- Fisher (2002) studies the equilibrium Repo rate
- Bottazzi et al. (2012) investigate the recursive use of securities as collateral, Rehypothecation
- Duffie & Krishnamurthy (2016) show how market frictions affects effectiveness of monetary policy
- Huh & Infante (2017) attribute specialness to collateral bid-ask spread
- Rahmouni-Rousseau & Vari (2020) suggest that specialness is associated to collateral scarcity

Borrowing rate

- Determined by the utilisation rate of a liquidity pool
- Utilisation rate: # of coins loaned out / total # of coins provided by lenders
- Mainstream: Klink algorithm (e.g. on AAVE, Compound):



- Continuously compounding
- Rate changes whenever there is a transaction
- Parameters are decided by the platform (baseline, optimal utilisation, slopes)

Borrowing rate

Idea

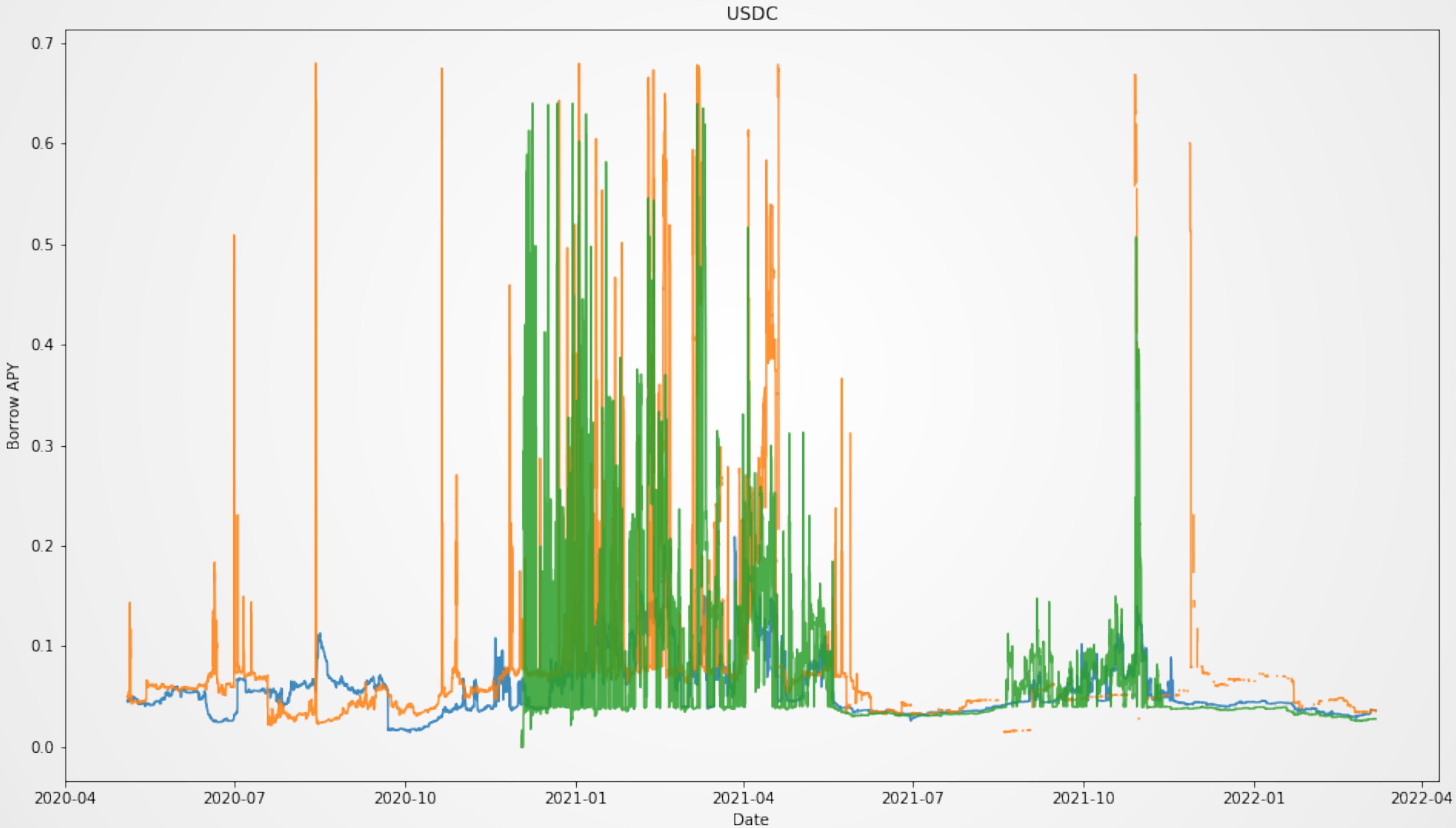
- Provides quick response to demand and supply
- High borrowing rate: attracts lenders and encourages borrowers to repay
- Low borrowing rate: attracts borrowers and encourages lenders to withdraw

Problems

- Not attached to fixed maturity $\succ$ Term structure is not available
- Solely determined by *liquidity-pool-specific* demand and supply $\succ$ Difficult to model
- Reflects a combination of risks (platform-specific + market risk) $\succ$ Difficult to hedge

Borrowing rate

Time series of USDC borrowing rate (annualised) in major platforms



Compound, **AAVE v1**, **AAVE v2**

ratesPlots

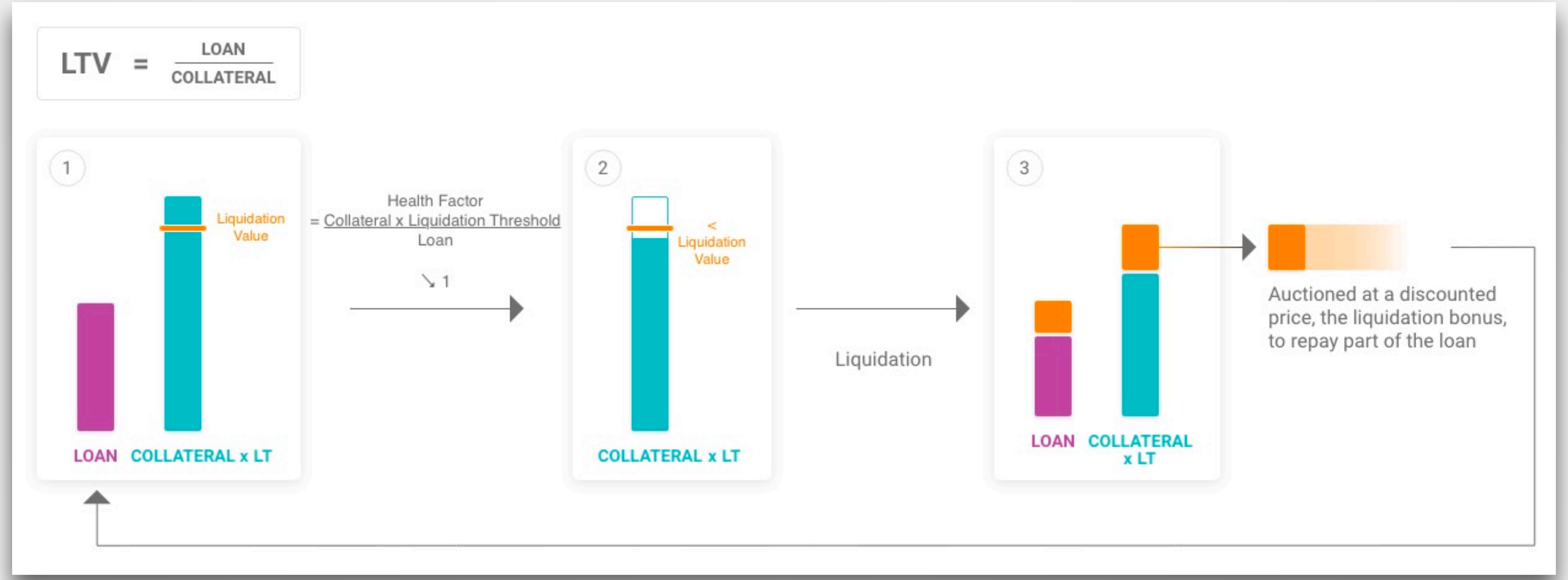
Liquidation

- Sells the collateral to liquidators at a discount (liquidation bonus) to repay the loan
- Ensures loans are always over-collateralised
- Triggered when borrowers' Loan-to-Value (LTV) reach a threshold LTV_H
- LTV:

$$\text{LTV}_t = \frac{\text{total debt at time } t}{\text{total collateral at time } t}$$

- Mostly triggered by sudden collateral price drops

Liquidation



AAVE

- Selling 50% of the collateral *at a discount* (~5%) to repay part of the outstanding debt
- Active loan position remains after liquidation
- Successive(cascading) liquidations is possible (example)

YouHodler

- Close borrower's position
- Sell *all* collateral to market to repay all the outstanding debt
- Return the remaining fund to borrower

The synthetic contract

A basic loan contract for analysis:

- The contract allows borrower to borrow USD against BTC collateral
- The initial collateral is LTV_0 USD worth BTC per USD borrowed
- The loan will be matured at time T ≻ Attach a maturity
- Borrower can repay at $t \in [\Delta t, T]$ ≻ $\Delta t > 0$ earliest unwinding time
- Borrowing rate is an *annualised continuously compounding rate*, risk-free rate r plus a premium κ

The synthetic contract

Liquidation setting ≻ YouHodler's liquidation

- Borrower has *no assess* to the collateral unless she repays all the outstanding debt.
- *All* the collateral will be liquidated if the position LTV is higher than $\underline{\mathrm{LTV}_H}$; Or the borrower does not ever repay until time T.
- Liquidation turns *all* the BTC collateral into USD at market price of the time. After repaying the debt and the accrued interest, the remaining USD will be returned to the borrower.

Borrower's strategy and payoff

Market

- At time 0, the price of BTC is S_0 USD

Strategy

- Pledge S_0 USD worth crypto as collateral ≻ 1 BTC collateral
- Borrow $S_0 \cdot \text{LTV}_0$ USD
- Repay the principal plus the accrued interests and sell the collateral to the market *whenever it is profitable*

If the position is unwinded at time t, the payoff in USD:

$$\begin{aligned} \phi_\kappa\left(S_t, t\right) &= \left(S_{t\wedge\tau} - \text{LTV}_0 S_0 e^{(r+\kappa)t\wedge\tau}\right)^+ \\ &= \left(S_{t\wedge\tau} - K e^{(r+\kappa)t\wedge\tau}\right)^+, \end{aligned}$$

where $t \wedge \tau = \min(t, \tau)$, $\tau = \inf\left\{t : \text{LTV}_t \geq \text{LTV}_H\right\}$ with $\inf\{\varnothing\} = \infty$.

≻ Barrier call option with exponentially growing strike and barrier (on next slide)

Extension: Chain up loans

Liquidation threshold

- Express the liquidation criteria in price of collateral
- Recall

$$\tau = \inf\{t : \mathrm{LTV}_t \geq \mathrm{LTV}_H\}$$

- Rewrite and rearrange

$$\begin{aligned}\tau &= \inf\left\{t : \frac{\mathrm{LTV}_0 S_0 e^{(r+\kappa)t}}{S_t} \geq \mathrm{LTV}_H\right\} \\ &= \inf\left\{t : S_t \leq \frac{\mathrm{LTV}_0}{\mathrm{LTV}_H} S_0 e^{(r+\kappa)t}\right\} \\ &= \inf\left\{t : S_t \leq H e^{(r+\kappa)t}\right\}\end{aligned}$$

$$K = \mathrm{LTV}_0 S_0$$

$$H = \frac{\mathrm{LTV}_0}{\mathrm{LTV}_H} S_0$$

≻ Barrier is always larger than strike (due to over-collateralisation $\mathrm{LTV}_H < 1$)

No-arbitrage price

- Suppose the uncertainty of the crypto collateral price is described by a filtered risk-neutral probability space $\left(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{Q}\right)$, the no-arbitrage price of the loan position is:

$$g(\kappa, \Delta t, T) = \sup_{\Delta t \leq t \leq T} \mathsf{E}_{\mathbb{Q}}\left[e^{-rt}\phi_\kappa\left(S_t, t\right)\right].$$

- American type: The supremum is taken over all stopping times between Δt and T
- Equivalent to long a American barrier option written on the collateral
- Cost of entering the position is the *haircut* of the loan:

$$S_0 - \mathsf{LTV}_0 S_0$$

- *Net cash flow* at contract inception:

$$-S_0 + \mathsf{LTV}_0 S_0 + g(\kappa, \Delta t, T)$$

Characteristics

Lemma 1. If $\kappa_1 > \kappa_2$, then $g(\kappa_1, \Delta t, T) < g(\kappa_2, \Delta t, T)$ for any $0 \leq \Delta t < T$ and $T > 0$.

≻ Lower the premium, higher the value of the borrowing position

Lemma 2. If $T_1 > T_2$, then $g(\kappa, \Delta t, T_1) \geq g(\kappa, \Delta t, T_2)$ for any $0 \leq \Delta t < T_2$ and $\kappa \in \mathbb{R}$.

≻ Longer the maturity, higher the value of the borrowing position

Lemma 3. $g(\kappa, \Delta t, T) \geq \mathsf{E}\left[\left(S_{\Delta t \wedge \tau} - \text{LTV}_0 S_0\right)^+ \middle| \mathscr{F}_0\right]$.

Corollary 1. $g(\kappa, 0, T) \geq S_0 - \text{LTV}_0 S_0$.

≻ If borrower can repay right at contract inception, the smallest position value is the *haircut*

≻ Borrower pays *zero* interest to lender (not an interesting case)

≻ Set $\Delta t > 0$ for further analysis

Proofs

Fair premium

- To avoid arbitrage (see Xia & Zhou (2007)), the fair borrowing premium $\hat{\kappa}$ is the premium that brings the net cash flow at contract inception to 0:

$$-S_0 + \text{LTV}_0 S_0 + g(\hat{\kappa}(T), \Delta t, T) = 0$$

- The term structure $\left\{\hat{\kappa}(t)\right\}_{t \geq \Delta t}$ is always *contango* as a result of lemma 1 and lemma 2

- $\hat{\kappa}(T)$s depend on the choice of risk neutral measure $\mathbb{Q}$

Theorem 1. If the discounted collateral price process is *continuous* under a risk-neutral measure, then the fair (arbitrage-free) borrowing premium of the synthetic contract is *zero*.

$\succ$ The positive borrowing premium observed from the market can be seen as a compensation to the *discontinuity* of the collateral price process or some other issues.

Proof of Theorem 1:

Since $K < H$ and $\left\{S_t\right\}_{0\leq t\leq T}$ is continuous (recall $\tau = \inf\left\{S_t : S_t \leq He^{(r+\kappa)t}\right\}$),

$$\left(S_{\tau\wedge T} - e^{(r+\kappa)(\tau\wedge T)}K\right)^+ = S_{\tau\wedge T} - e^{(r+\kappa)(\tau\wedge T)}K.$$

By optional stopping theorem, the follow holds for any stopping time $\tau \wedge T$

$$\mathsf{E}\left[e^{-r(\tau\wedge T)}\left(S_{\tau\wedge T} - e^{(r+\kappa)(\tau\wedge T)}K\right)\right] = S_0 - K\cdot\mathsf{E}\left(e^{\kappa(\tau\wedge T)}\right).$$

Therefore,

$$\begin{aligned} g(\kappa, \Delta t, T) &= \sup_{\Delta t\leq t\leq T} \mathsf{E}\left[e^{-r(\tau\wedge t)}\left(S_{\tau\wedge t} - e^{(r+\kappa)(\tau\wedge t)}K\right)\right] \\ &= S_0 - K \sup_{\Delta t\leq t\leq T}\left[\mathsf{E}\left(e^{\kappa(\tau\wedge t)}\right)\right]. \end{aligned}$$

As the consequence, for $-S_0 + K + g(\hat{\kappa}, \Delta t, T) = 0$ (no-arbitrage/zero net cashflow at inception) to hold, $\hat{\kappa}$ must be zero.

Numerical procedure

- To find $\hat{\kappa}$:
  1. Obtain a set of $g(\kappa, \Delta t, T)$s from a pricer for an array of κ_is (Longstaff and Schwartz)
  2. Get an approximation $\tilde{g}(\kappa), \forall \kappa \in \mathbb{R}$ by polynomial interpolation
  3. Set $\hat{\kappa} = \tilde{g}^{(-1)}(S - K)$
- Further inspect the relationship between $\hat{\kappa}$ and T by repeat the procedure with different maturities T
- Form a fair premium curve (analogous to yield curves)

Collateral price process

Double Exponential Jump Process (Kou, 2002)

- Allow asymmetric jump sizes
- Able to fit crypto IV surface nicely
- Popular choice of discontinuous price process

Poisson process with intensity λ

Volatility

$$\frac{dS_t}{S_{t-}} = \left(r - \lambda\zeta\right) dt + \sigma dW_t + d\left\{\sum_{i=1}^{N_t}(V_i - 1)\right\}, \zeta = \mathrm{E}(V),$$

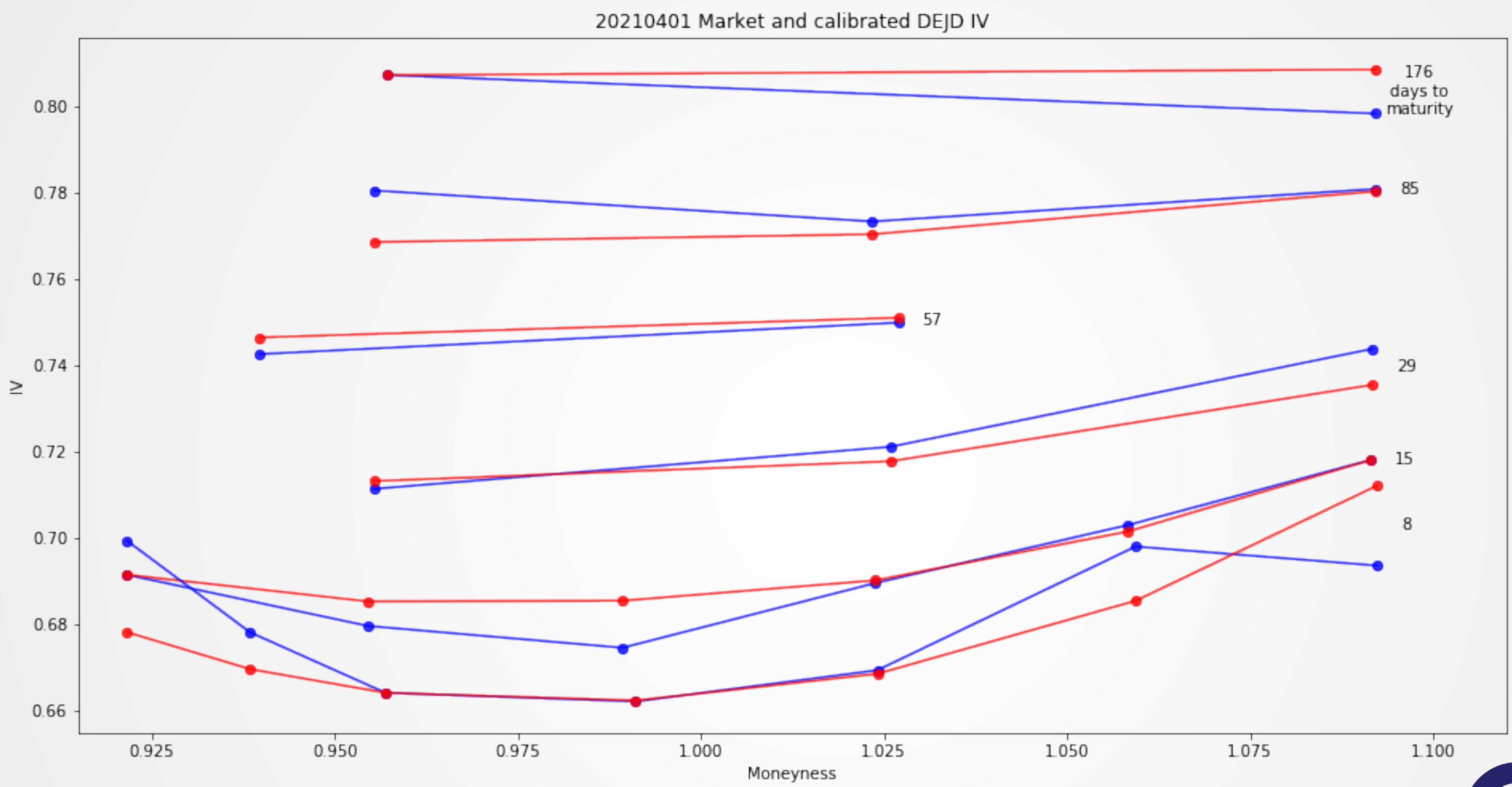
$$Y = \log(V),\ f_Y(y) = p\eta_1 e^{-\eta_1 y} \cdot 1(y \geq 0) + (1-p)\eta_2 e^{-\eta_2 y} \cdot 1(y < 0)$$

Probability of a positive jump

Parameter for positive jump size

Parameter for negative jump size

Deribit ETH IV on 20210401

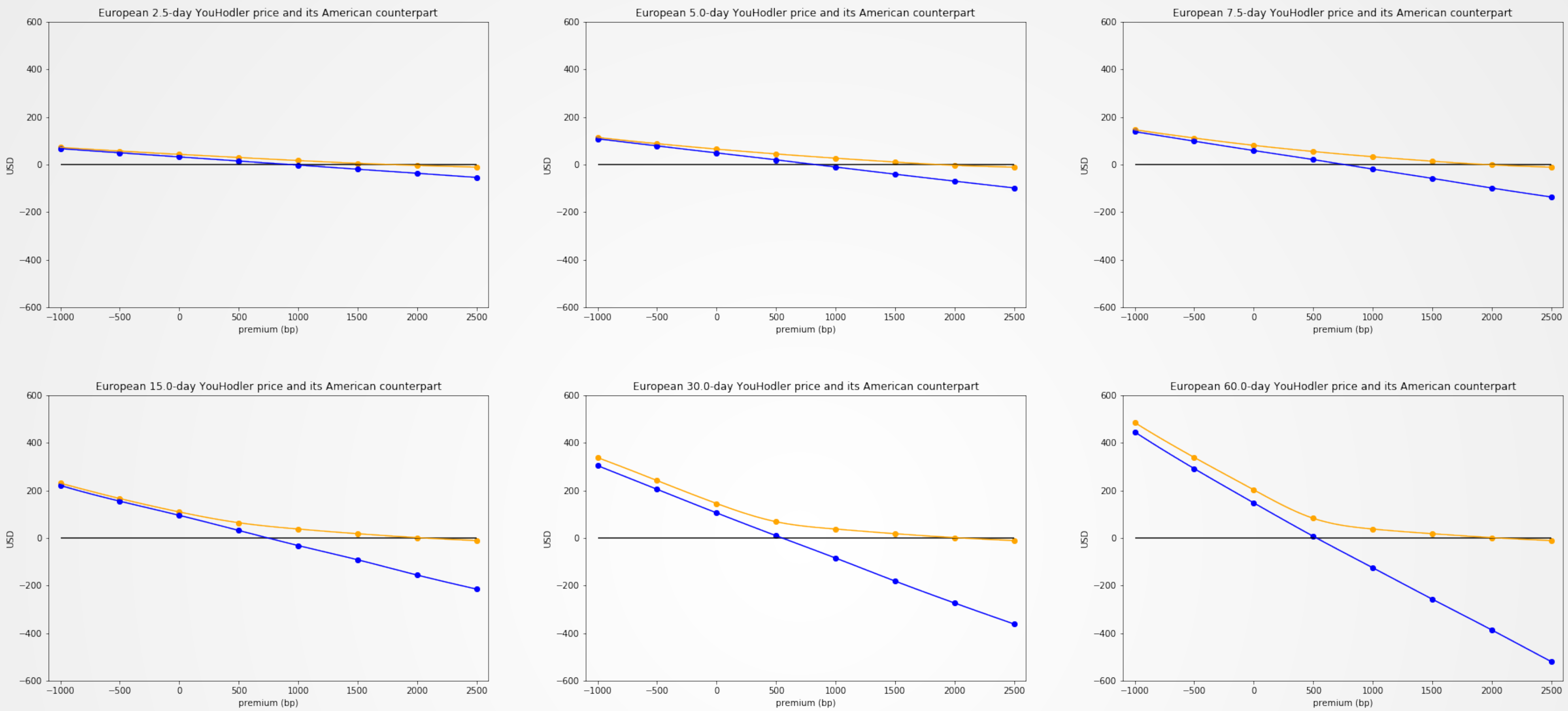


Market IV; **fitted DEJD IV**

$$\hat{\sigma} = .59,\ \hat{\lambda} = .95,\ 1/\hat{\eta}_1 = .43,\ 1/\hat{\eta}_2 = .48,\ \hat{p} = .46$$

fit_IV_DEJD

Results from pricing algo

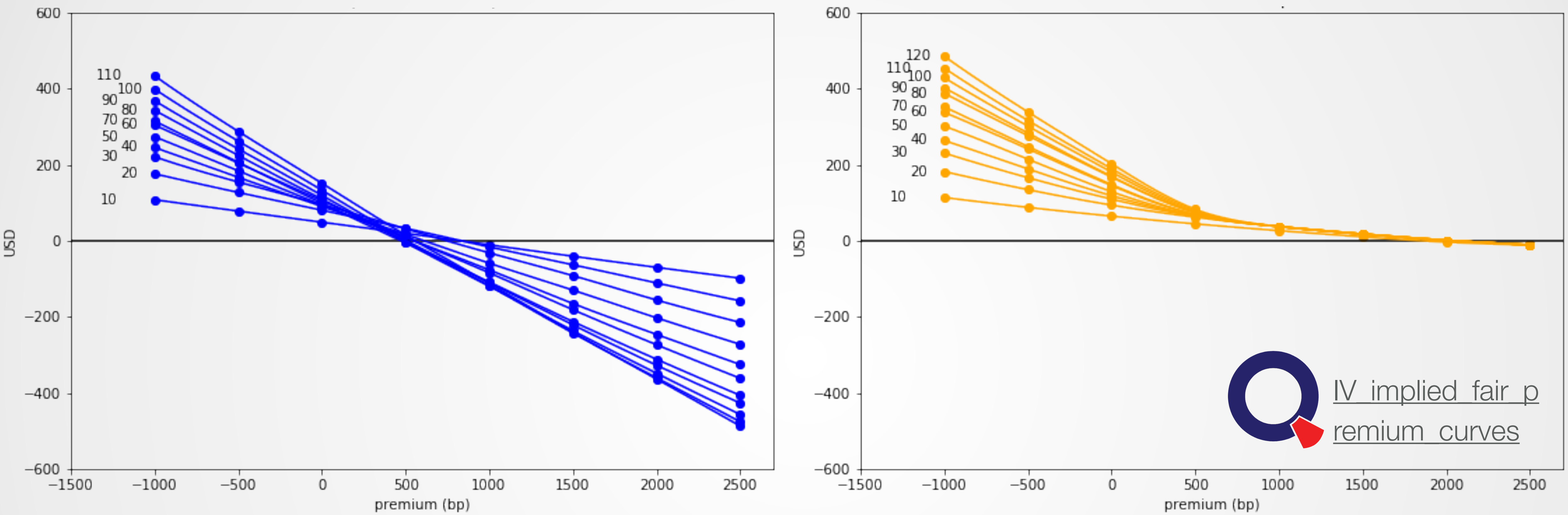


European net cash flow (EU); **American net cash flow (AM)**

[IV_implied_fair_premium_curves]

- AM > EU; Downward slopping prices;
- When the borrowing rate is higher (left to right on each panel)
  - Gap between the AM and EU widens
- When maturity increases (panels from left to right, top to bottom)
  - Spread between the two prices increases (see also next slide)
  - Ams shift upward on the left; Slope decreases (see also next slide)
  - EUs' slope decreases

Results from pricing algo



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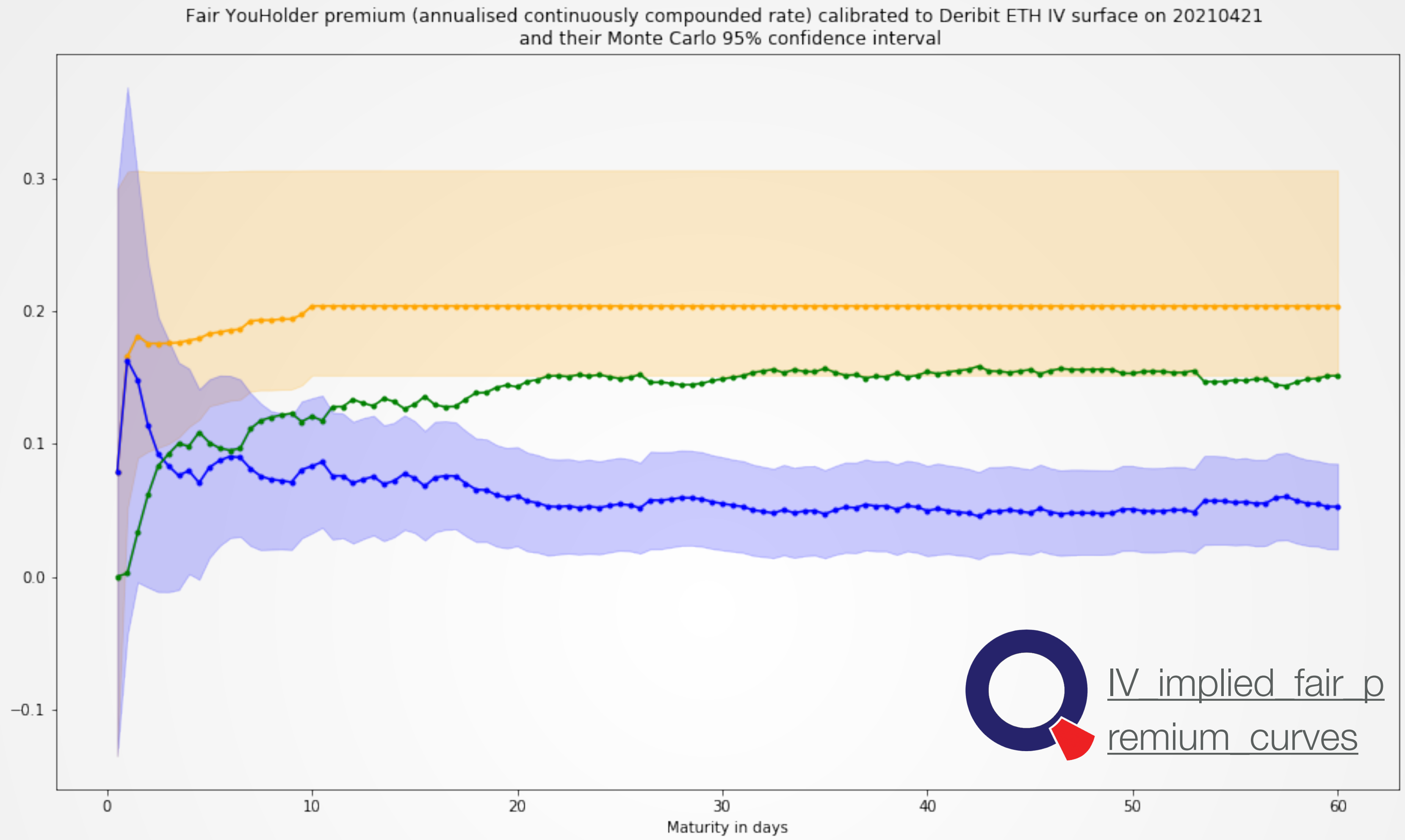
Term structure



EU fair premium, **AM fair premium**, **Early stopping premium**

- **EU**
  - Large CI covering negative values when T is small,
  - Hump at $T = 1$ days; decreasing afterwards
  - Rough but converging; converge to **5.3%**
- **AM**
  - Non-decreasing function of maturity
  - consistent CI that does not cover negative values after $T = 10$
  - Level off after $T = 10$ days and converge to **20.4%**
- **Early stopping premium**
  - AM-EU
  - Roughly a non-decreasing function of maturity
  - 0 when maturity is short, increasing trend

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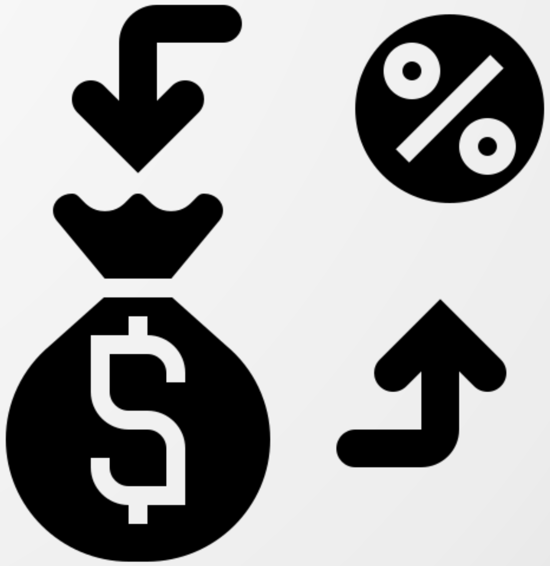


Regression-based pricing algorithm

Longstaff and Schwartz (2001) method:

1. Simulate b independent paths $\left\{S_{1j}, \ldots, S_{mj}\right\}, j = 1,\ldots,b$
2. At terminal nodes, set $\hat{V}_{mj} = \phi_\kappa\left(S_{mj}\right)$
3. Apply backward induction: for $i = m - 1,\ldots,1$,
   i) Fit regression $\hat{V}_{i+1,j} = \hat{C}_i\left(S_i\right) + \varepsilon$
   ii) Set $\hat{V}_{ij} = \begin{cases} \phi_\kappa(S_{ij}) & \text{, if } \phi_\kappa(S_{ij}) \geq \hat{C}(S_{ij}) \\ \hat{V}_{i+1,j} & \text{, else.} \end{cases}$, for $j = 1,\ldots,b$
3. Set $\hat{V}_0 = \left(\hat{V}_{11} + , \ldots, \hat{V}_{1b}\right) \Big/ b$

See also Glasserman (2004), and Tsitsiklis and Van Roy (2001).

Proofs

Proof of Lemma 1.

Let $\tau_1 = \inf\left\{t : S_t \leq \frac{\mathsf{LTV}_0}{\mathsf{LTV}_H} S_0 e^{(r+\kappa_1)t}\right\}$ and $\tau_2 = \inf\left\{t : S_t \leq \frac{\mathsf{LTV}_0}{\mathsf{LTV}_H} S_0 e^{(r+\kappa_2)t}\right\}$ be the liquidation times.

Since $\tau_1 \leq \tau_2$, i.e. collateral price must pass through a higher barrier from above before passing through a lower barrier from above,

$$\left\{\mathsf{E}\left[\left(S_{t\wedge\tau_1} - \mathsf{LTV}_0 S_0 e^{(r+\kappa_1)(t\wedge\tau_1)}\right)^+\right]\right\}_{\Delta t \leq t \leq T} \subseteq \left\{\mathsf{E}\left[\left(S_{t\wedge\tau_2} - \mathsf{LTV}_0 S_0 e^{(r+\kappa_1)(t\wedge\tau_2)}\right)^+\right]\right\}_{\Delta t \leq t \leq T}.$$

Therefore,

$$\sup_{\Delta t \leq t \leq T} \mathsf{E}\left[\left(S_{t\wedge\tau_1} - \mathsf{LTV}_0 S_0 e^{(r+\kappa_1)(t\wedge\tau_1)}\right)^+\right] \leq \sup_{\Delta t \leq t \leq T} \mathsf{E}\left[\left(S_{t\wedge\tau_2} - \mathsf{LTV}_0 S_0 e^{(r+\kappa_1)(t\wedge\tau_2)}\right)^+\right].$$

Since $\kappa_1 > \kappa_2$

$$\sup_{\Delta t \leq t \leq T} \mathsf{E}\left[\left(S_{t\wedge\tau_2} - \mathsf{LTV}_0 S_0 e^{(r+\kappa_1)(t\wedge\tau_2)}\right)^+\right] < \sup_{\Delta t \leq t \leq T} \mathsf{E}\left[\left(S_{t\wedge\tau_2} - \mathsf{LTV}_0 S_0 e^{(r+\kappa_2)(t\wedge\tau_2)}\right)^+\right].$$

Proofs

Proof of Lemma 2.

Since $T_1 > T_2$,

$$\left\{ \mathsf{E}\left[\left(S_{t\wedge\tau} - \mathsf{LTV}_0 S_0 e^{(r+\kappa)(t\wedge\tau)} \right)^+ \right] \right\}_{\Delta t \leq t \leq T_1} \supseteq \left\{ \mathsf{E}\left[\left(S_{t\wedge\tau} - \mathsf{LTV}_0 S_0 e^{(r+\kappa_1)(t\wedge\tau)} \right)^+ \right] \right\}_{\Delta t \leq t \leq T_2}.$$

Therefore,

$$\sup_{\Delta t \leq t \leq T_1} \mathsf{E}\left[\left(S_{t\wedge\tau} - \mathsf{LTV}_0 S_0 e^{(r+\kappa)(t\wedge\tau)} \right)^+ \right] \geq \sup_{\Delta t \leq t \leq T_2} \mathsf{E}\left[\left(S_{t\wedge\tau} - \mathsf{LTV}_0 S_0 e^{(r+\kappa)(t\wedge\tau)} \right)^+ \right].$$

Proofs

Proof of Lemma 3.

Since $\{t = \Delta t\} \in \mathscr{F}_0$ is a stopping time,

$$\mathsf{E}\left[\left(S_{\Delta t \wedge \tau} - \mathsf{LTV}_0 S_0\right)^+\right] \in \left\{\mathsf{E}\left[\left(S_{t \wedge \tau} - \mathsf{LTV}_0 S_0 e^{(r+\kappa)(t\wedge\tau)}\right)^+\right]\right\}_{\Delta t \le t \le T}.$$

Therefore,

$$\sup_{\Delta t \le t \le T} \mathsf{E}\left[\left(S_{t \wedge \tau} - \mathsf{LTV}_0 S_0 e^{(r+\kappa)(t\wedge\tau)}\right)^+\right] \ge \mathsf{E}\left[\left(S_{\Delta t \wedge \tau} - \mathsf{LTV}_0 S_0\right)^+\right].$$

Example 1: Back borrow by multiple collateral types

WETH Collateral

	timestamp	value	event
0	2022-03-10 18:10:49	2.4	enableAsCollateral
1	2022-05-27 05:48:03	2.85075	depositAsCollateral
2	2022-06-13 07:57:50	1.47886	liquidationCall
3	2022-06-15 04:28:08	0.692964	liquidationCall
4	2022-06-18 09:46:59	0.23185	liquidationCall

ENJ Collateral

	timestamp	value	event
0	2022-03-10 18:50:22	900	enableAsCollateral

DAI Borrow

	timestamp	value	event
0	2022-03-10 18:56:35	3299	borrow
1	2022-06-13 07:57:50	1616.94	liquidationCall
2	2022-06-15 04:28:08	767.005	liquidationCall
3	2022-06-18 09:46:59	333.453	liquidationCall

User id: 0x07f23457d4282e3119244a62448483357ee25cf5

Cascading liquidation calls

Liquidators choose which collateral to liquidate. In this case, they chose WETH.

Example 2: Multiple borrows multiple collateral types

WETH Borrow

	timestamp	value	event
0	2021-01-16 05:53:00	124.45	borrow
1	2021-01-16 06:47:25	494.45	borrow
2	2021-01-16 06:54:38	554.45	borrow
3	2021-01-20 04:06:32	0	repay
4	2021-01-20 21:01:49	124.823	borrow
5	2021-01-20 22:44:33	53.8234	repay
6	2021-01-22 05:46:58	0	repay
7	2021-01-31 07:41:27	943.82	borrow
8	2021-01-31 16:27:21	713.82	repay
9	2021-02-02 06:31:22	863.82	borrow
10	2021-02-03 05:34:09	1199.82	borrow
11	2021-02-04 06:05:44	1529.82	borrow
12	2021-02-05 02:18:56	2095.82	borrow
13	2021-02-11 05:49:02	995.82	repay
14	2021-02-11 05:55:34	0	repay
15	2021-02-19 01:40:00	198.51	borrow
16	2021-02-24 01:10:05	598.51	borrow
17	2021-03-07 15:20:58	0	repay
18	2021-03-08 22:41:34	223.174	borrow
19	2021-03-27 04:28:33	203.174	repay
20	2021-04-07 06:06:16	147.174	repay
21	2021-04-18 20:41:40	0	repay
22	2021-04-18 20:58:25	43.807	borrow
23	2021-04-19 03:27:43	223.807	borrow
24	2021-04-22 05:15:26	0	repay

DAI Borrow

	timestamp	value	event
0	2021-01-20 04:56:50	2.358e+06	borrow
1	2021-01-21 12:47:17	1.91629e+06	repay
2	2021-01-24 21:44:18	3.21629e+06	borrow
3	2021-02-04 05:50:34	3.81629e+06	borrow
4	2021-02-24 05:07:49	379642	repay
5	2021-04-22 05:12:36	0	repay
6	2021-05-14 20:49:40	1.21505e+06	borrow
7	2021-06-04 06:35:09	2.21505e+06	borrow
8	2021-06-04 19:38:33	963574	repay
9	2021-06-04 19:55:05	1.23573e+06	borrow
10	2021-06-04 20:51:23	963871	repay
11	2021-06-18 05:24:54	0	repay

LINK Borrow

	timestamp	value	event
0	2021-03-07 06:20:43	200000	borrow
1	2021-03-07 15:24:28	290000	borrow
2	2021-03-07 20:52:45	330000	borrow
3	2021-03-08 22:20:26	0	repay
4	2021-03-08 22:29:54	0	repay

USDC Borrow

	timestamp	value	event
0	2021-01-16 05:56:01	1e+06	borrow
1	2021-01-16 06:06:27	1.69576e+06	borrow
2	2021-01-16 06:24:23	995639	repay
3	2021-01-16 06:35:16	0	repay
4	2021-01-23 04:01:24	359976	borrow
5	2021-01-30 15:14:31	659976	borrow
6	2021-01-31 07:32:45	2.15998e+06	borrow
7	2021-01-31 16:27:21	1.8549e+06	repay
8	2021-02-14 06:46:15	2.0566e+06	borrow
9	2021-02-23 09:43:47	1.0998e+06	repay
10	2021-02-24 05:19:23	0	repay
11	2021-04-18 20:54:23	585514	borrow
12	2021-04-19 03:23:34	0	repay
13	2021-04-23 03:54:03	1.68548e+06	borrow
14	2021-04-23 04:01:40	3.18548e+06	borrow
15	2021-05-01 21:38:53	3.78548e+06	borrow
16	2021-05-06 04:50:27	2.44478e+06	repay
17	2021-05-12 05:33:49	2.78403e+06	borrow
18	2021-06-01 21:03:46	1.76287e+06	repay
19	2021-06-04 19:52:37	4.49014e+06	borrow
20	2021-06-04 20:51:29	1.76287e+06	repay
21	2021-06-12 22:11:55	1.6914e+06	repay
22	2021-06-13 21:07:58	1.18792e+06	repay
23	2021-06-13 21:40:46	0	repay

TUSD Borrow

	timestamp	value	event
0	2021-02-02 06:14:43	250000	borrow
1	2021-02-24 05:11:19	94555.4	repay
2	2021-04-22 05:06:54	0	repay

SUSD Borrow

	timestamp	value	event
0	2021-05-14 20:53:13	1.25e+06	borrow
1	2021-05-14 21:14:25	174940	repay
2	2021-06-18 05:24:54	0	repay

USDT Borrow

	timestamp	value	event
0	2021-02-03 05:30:16	600000	borrow
1	2021-02-05 02:18:56	0	repay
2	2021-03-14 04:49:04	89465.5	borrow
3	2021-03-14 04:59:36	179465	borrow
4	2021-03-14 05:07:39	89465.5	repay
5	2021-04-17 06:12:41	0	repay
6	2021-04-23 04:38:16	5.29823e+06	borrow
7	2021-04-23 04:58:05	5.99823e+06	borrow
8	2021-05-06 04:55:32	7.99823e+06	borrow
9	2021-05-14 21:03:04	9.19823e+06	borrow
10	2021-05-16 20:54:46	7.78312e+06	repay
11	2021-05-19 04:59:32	5.97676e+06	repay
12	2021-05-19 13:20:23	4.94508e+06	repay
13	2021-06-02 06:00:31	4.95508e+06	borrow
14	2021-06-04 06:23:40	5.95508e+06	borrow
15	2021-06-04 19:46:02	5.19971e+06	repay
16	2021-06-04 21:51:44	7.37193e+06	borrow
17	2021-06-12 22:12:17	7.35682e+06	repay
18	2021-06-13 21:13:06	3.82736e+06	repay
19	2021-06-13 21:39:51	8.05526e+06	borrow
20	2021-06-14 19:22:53	7.43654e+06	repay
21	2021-06-18 05:24:54	0	repay

0x057518153ed7f25dd237a0d0052ae8cc5c428ee3

Example 2: Multiple borrows multiple collateral types

WBTC Collateral

	timestamp	value	event
0	2021-01-16 05:50:19	62.5	enableAsCollateral
1	2021-01-16 06:04:28	117.5	depositAsCollateral
2	2021-01-16 06:27:36	57.5	redeemUnderlying
3	2021-01-16 06:42:56	27.5	redeemUnderlying
4	2021-01-16 06:53:31	36.2382	depositAsCollateral
5	2021-01-20 04:55:38	160.199	depositAsCollateral
6	2021-01-20 05:52:06	118.199	redeemUnderlying
7	2021-01-24 21:40:19	190.373	depositAsCollateral
8	2021-01-31 07:30:41	311.525	depositAsCollateral
9	2021-02-02 01:38:42	361.423	depositAsCollateral
10	2021-02-03 06:13:52	400.674	depositAsCollateral
11	2021-02-11 05:43:06	325.674	redeemUnderlying
12	2021-02-11 05:52:39	260.674	redeemUnderlying
13	2021-02-11 05:57:42	224.978	redeemUnderlying
14	2021-02-19 01:23:49	214.978	redeemUnderlying
15	2021-02-24 05:27:49	47.978	redeemUnderlying
16	2021-03-07 06:17:01	192.678	depositAsCollateral
17	2021-03-08 22:23:23	120.678	redeemUnderlying
18	2021-03-08 22:41:34	27.978	redeemUnderlying
19	2021-04-18 20:53:43	39.978	depositAsCollateral
20	2021-04-19 03:30:37	28.978	redeemUnderlying
21	2021-04-22 05:17:20	0	disableAsCollateral
22	2021-04-23 03:51:44	0	enableAsCollateral
23	2021-04-23 03:51:44	244.969	depositAsCollateral
24	2021-04-23 03:58:55	313.713	depositAsCollateral
25	2021-04-23 04:47:20	341.767	depositAsCollateral
26	2021-05-01 21:37:47	326.767	redeemUnderlying
27	2021-05-17 05:33:11	268.767	redeemUnderlying
28	2021-05-22 17:57:34	228.767	redeemUnderlying
29	2021-05-23 17:09:13	300.451	depositAsCollateral
30	2021-05-23 18:13:38	390.422	depositAsCollateral
31	2021-05-26 23:01:29	290.422	redeemUnderlying
32	2021-06-04 06:23:09	294.069	depositAsCollateral
33	2021-06-04 06:45:09	321.405	depositAsCollateral
34	2021-06-04 21:45:33	394.426	depositAsCollateral
35	2021-06-13 21:07:58	334.426	redeemUnderlying
36	2021-06-13 21:13:05	335.216	depositAsCollateral
37	2021-06-18 05:24:54	0	disableAsCollateral

WETH Collateral

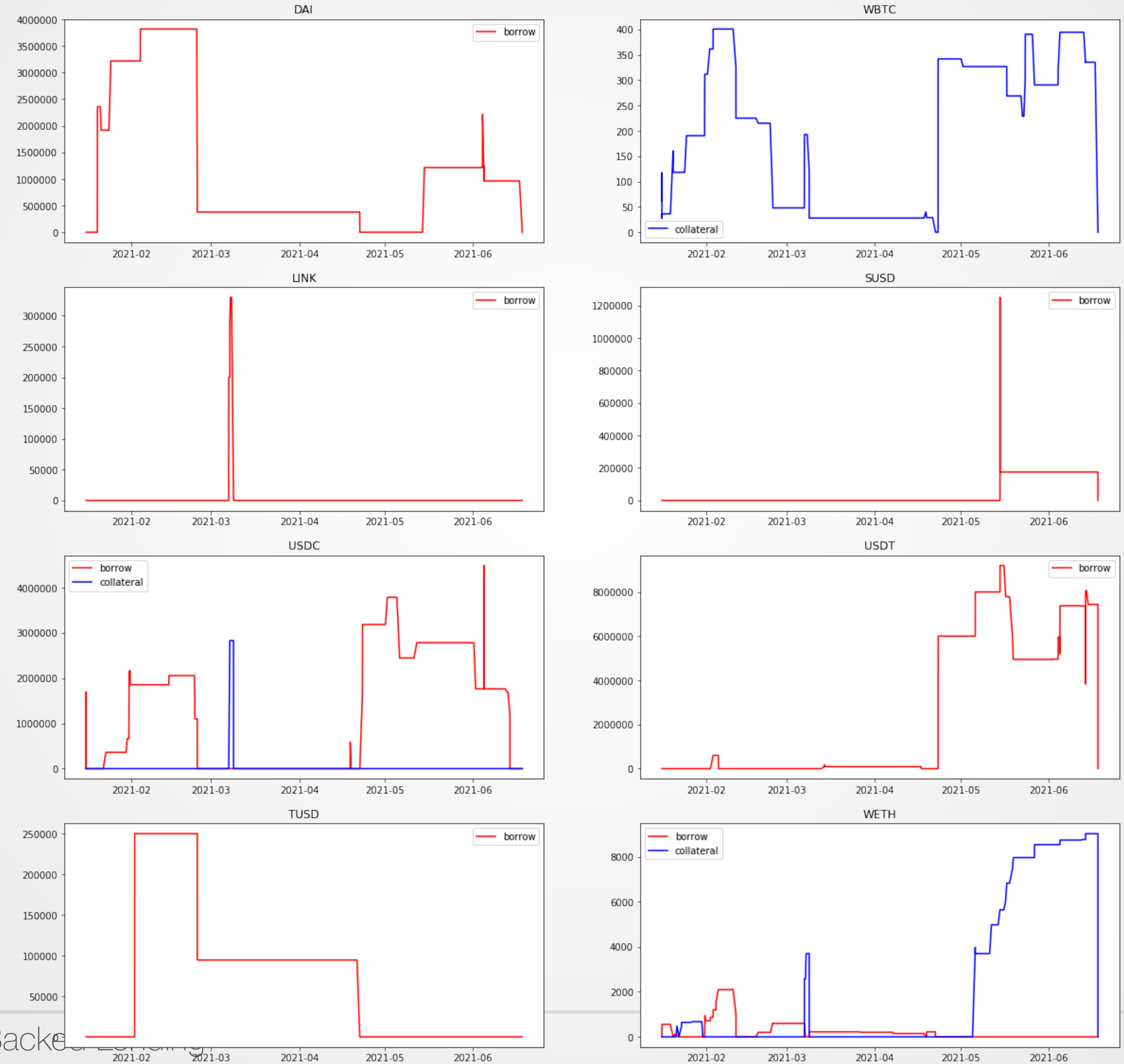
	timestamp	value	event
0	2021-01-21 12:50:29	480	depositAsCollateral
1	2021-01-22 05:41:52	0	disableAsCollateral
2	2021-01-23 04:09:11	480	enableAsCollateral
3	2021-01-23 04:09:11	640	depositAsCollateral
4	2021-01-27 03:50:26	676	depositAsCollateral
5	2021-01-30 15:28:44	0	disableAsCollateral
6	2021-03-07 06:17:44	2576	enableAsCollateral
7	2021-03-07 20:49:14	3696	depositAsCollateral
8	2021-03-08 22:24:29	0	disableAsCollateral
9	2021-05-06 04:59:15	3696	enableAsCollateral
10	2021-05-06 04:59:15	3976	depositAsCollateral
11	2021-05-11 20:53:22	4976	depositAsCollateral
12	2021-05-14 20:49:40	5641	depositAsCollateral
13	2021-05-16 20:56:30	6061	depositAsCollateral
14	2021-05-17 05:32:03	6823	depositAsCollateral
15	2021-05-19 05:00:06	7448	depositAsCollateral
16	2021-05-19 13:22:17	7958	depositAsCollateral
17	2021-05-26 23:08:04	8390	depositAsCollateral
18	2021-05-27 00:27:49	8535.7	depositAsCollateral
19	2021-06-04 21:52:20	8735.7	depositAsCollateral
20	2021-06-12 22:12:27	8760.7	depositAsCollateral
21	2021-06-13 21:17:37	9020.7	depositAsCollateral
22	2021-06-18 05:24:54	0	disableAsCollateral

USDC Collateral

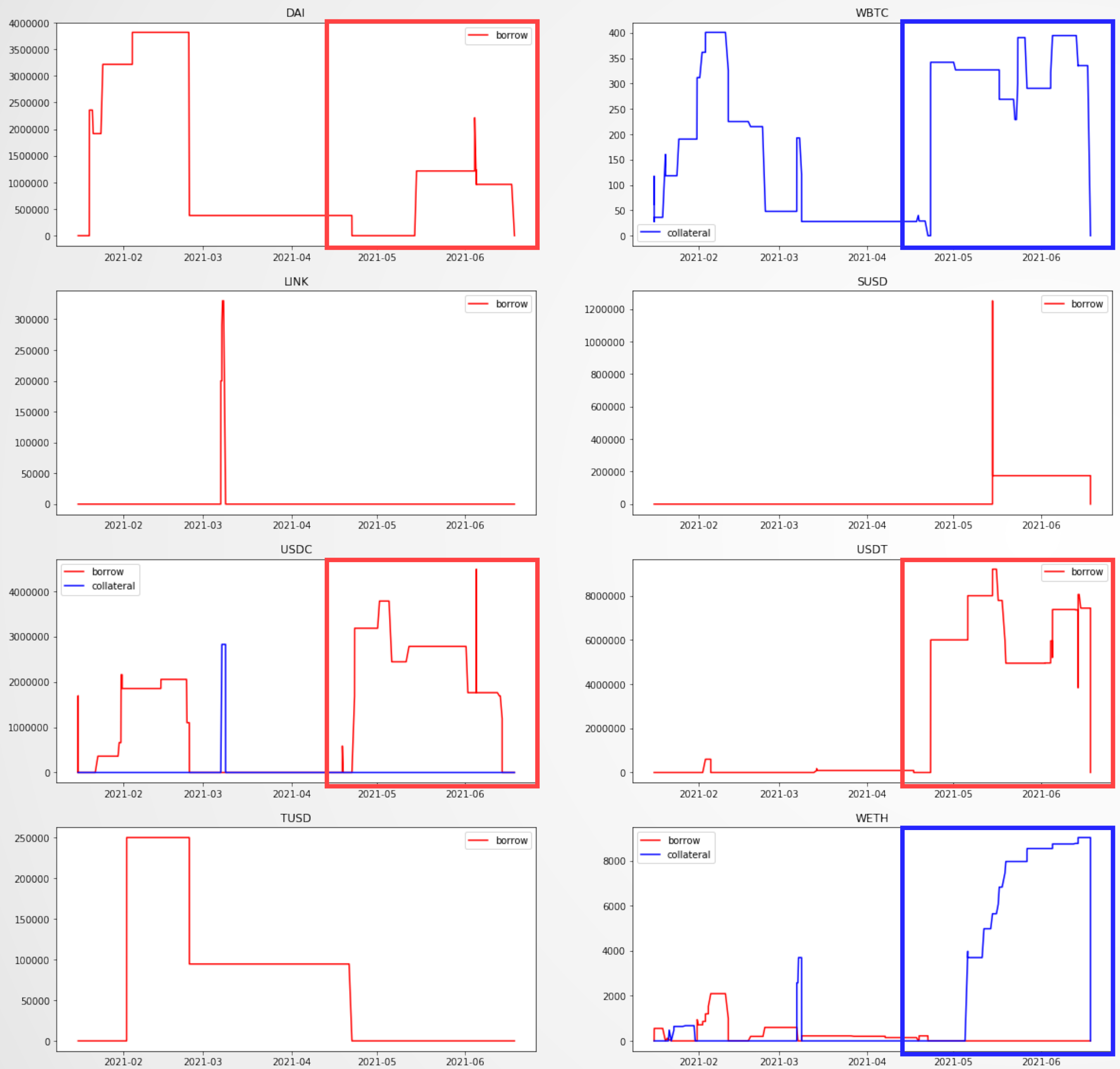
	timestamp	value	event
0	2021-03-07 15:23:00	2.82893e+06	depositAsCollateral
1	2021-03-08 22:24:02	0	disableAsCollateral

0x057518153ed7f25dd237a0d0052ae8cc5c428ee3

Example 2: Multiple borrows multiple collateral types



Example 2: Multiple borrows multiple collateral types



From 10-04-2021 to 19-6-2021

Collateral: WBTC, WETH

Borrow: USDT, DAI, USDC

⟹ short USD

Remarks

1. USDT, DAI, and USDC are stable coins pegged to USD
2. WBTC and WETH are wrapped version of BTC and ETH, one can think of them as BTC and ETH because they are 1:1 backed by BTC and ETH.

Chain up loans - Two-period model

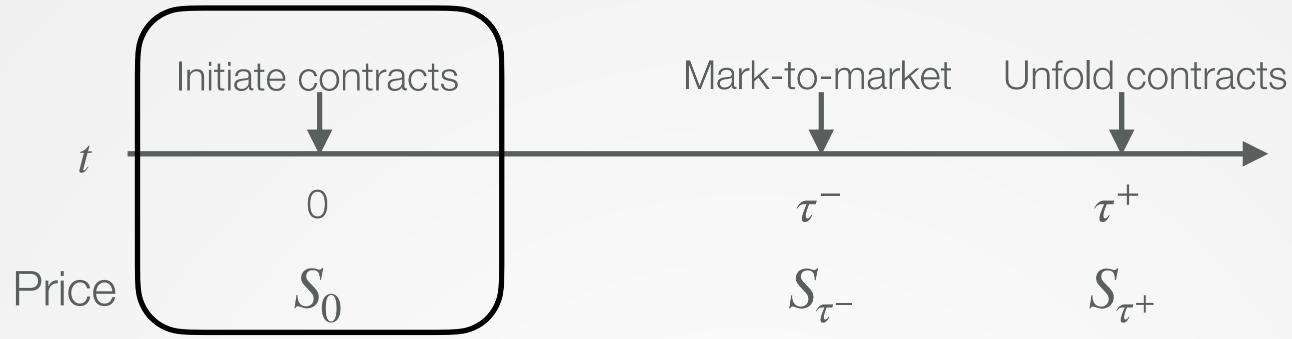
Assumptions

- Continuous price process, zero transaction cost (to be relaxed)
- Constant borrowing rate over time
- Borrower only act at the end of the timespan; Liquidator always liquidate half of the collateral

Borrowers

- At $t = 0$, Initiate multiple contracts: with 1 USD as initial investment, chain up contracts for F times
- At Mark-to-market τ^-: system checks: $\text{LTV}_t \geq \text{LTV}_H$
- Right after Mark-to-market τ^+: Borrower makes a decision whether to unfold the contracts (repay all debt and regain possession of collateral)

Chain up loans - Two-period model



At $t = 0$:

The total collateral is

$$C_0 = 1 + \text{LTV}_0 + \text{LTV}_0^2 + \text{LTV}_0^3 + \ldots + \text{LTV}_0^{F-1}$$
$$= \frac{1 - \text{LTV}_0^F}{1 - \text{LTV}_0}$$

The total number of coins in contracts is

$$N_0 = \frac{C_0}{S_0}$$

The total outstanding debt at is

$$O_0 = \text{LTV}_0 + \text{LTV}_0^2 + \text{LTV}_0^3 + \ldots + \text{LTV}_0^F$$
$$= \text{LTV}_0 \left(\frac{1 - \text{LTV}_0^F}{1 - \text{LTV}_0} \right)$$

So the total Loan-to-Value Ratio is

$$\frac{O_0}{C_0} = \text{LTV}_0$$

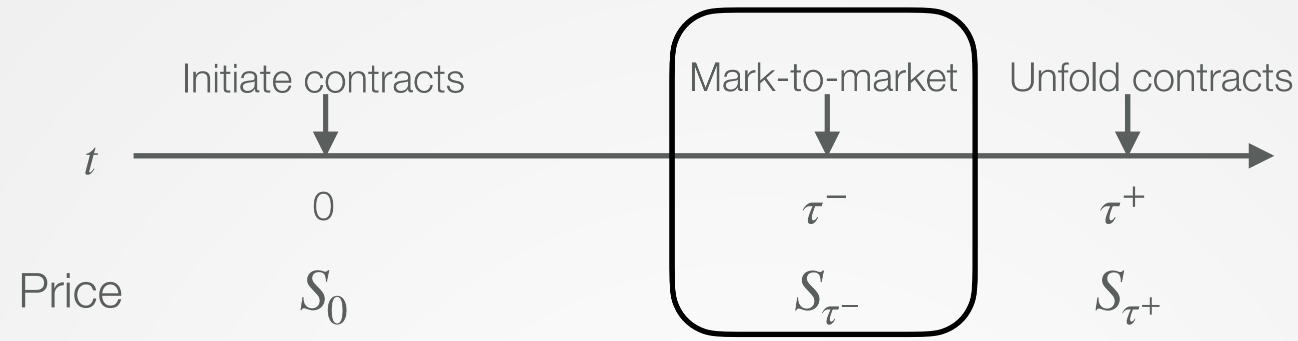
Cash Remaining: LTV_0^F

Observations:

Number of contracts does not affect the overall LTV;

Cost of entering this position is $1 - \text{LTV}_0^F$

Chain up loans - Two-period model



<u>At $t = \tau^-$ (before mark-to-market):</u>

The total outstanding debt at is

$$O_{\tau^-} = O_0 \cdot e^{(r+\kappa)\tau^-} \longleftarrow$$

r: risk-free rate
κ: premium to be determined

The total collateral is

$$C_{\tau^-} = N_0 \cdot S_{\tau^-}$$

The Loan-to-Value Ratio is

$$\text{LTV}_{\tau^-} = \frac{O_{\tau^-}}{C_{\tau^-}} = \frac{\text{LTV}_0 \cdot e^{(r+\kappa)\tau^-} \cdot S_0}{S_{\tau^-}}$$

<u>Observation</u>: LTV is a function of price and time

<u>At $t = \tau^+$ (after mark-to-market):</u>

The total outstanding debt at is

Number of coins to be liquidated

$$O_{\tau^+} = O_{\tau^-} - 1(\text{LTV}_{\tau^-} > \text{LTV}_H) \cdot N_1^l \cdot S_{\tau^-} \cdot \text{LB}$$

The total collateral is

$$C_{\tau^+} = \left\{N_0 - 1(\text{LTV}_{\tau^-} > \text{LTV}_H) \cdot N_1^l\right\} \cdot S_{\tau^+}$$

<u>Borrower's payoff:</u>

$$\text{Payoff} = (C_{\tau^+} - O_{\tau^+})$$

Borrowers only unfold the position if it generates positive cash flow

$$\text{Payoff} = (C_{\tau^+} - O_{\tau^+})^+ \longleftarrow$$ Call option payoff surfaces!

Chain up loans - Two-period model

<u>If $\text{LTV}_{\tau^-} < \text{LTV}_K$ (no liquidation)</u>

$$\text{Payoff}(S_{\tau^+}) = (C_{\tau^+} - O_{\tau^+})^+$$

$$= N_0 \left(S_{\tau^+} - \text{LTV}_0 S_0 e^{(r+\kappa)\tau^+} \right)^+$$ ← Factor out the number of coins as collateral N_0

$$= N_0 \left(S_{\tau^+} - K_0 e^{(r+\kappa)\tau^+} \right)^+$$ ← $K_0 \overset{\text{def}}{=} \text{LTV}_0 S_0$

<u>If $\text{LTV}_{\tau^-} \geq \text{LTV}_K$ (one liquidation)</u>

$$\text{Payoff}(S_{\tau^+}) = (C_{\tau^+} - O_{\tau^+})^+$$ ← Borrower remains loan position after one liquidation

$$= N_1 \left(S_{\tau^+} - K_1 e^{(r+\kappa)\tau^+} \right)^+$$ ← With new number of coins as collateral and strike

where $N_1 = N_0 - N_1^l$ is number of coins remaining after liquidation, and $K_1 = K_0 \dfrac{\text{LB}}{2\text{LB} - \text{LTV}_H}$; Proof.

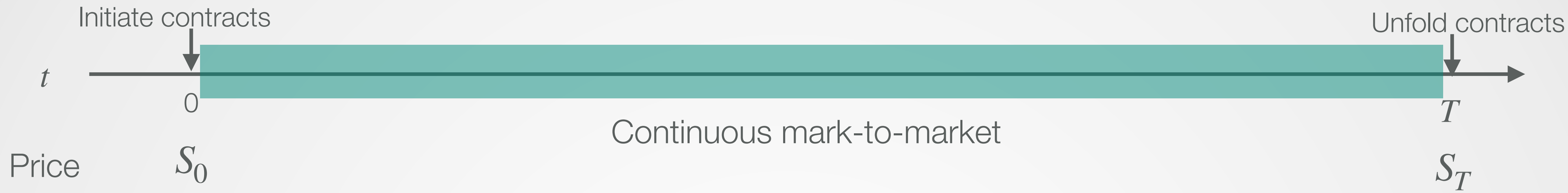
<u>Observations:</u>

Payoff looks like an call option with a constant multiplier with exponentially growing strike at rate $r + \kappa$

Borrower remains loan position after liquidation

Number of coins N_1 and strike K_1 after liquidation *do not* depend on liquidation time τ^- (if the price is continuous)

Chain up loans - Multi-period extension



Loan position remains a loan position after liquidation

- Consider multiple liquidations in $[0,T)$.
- The i^{th} liquidation is triggered at τ_i, where $\tau_1 < \tau_2 < \tau_3 < \ldots < T$.
- At time T, the outstanding debt is

$$O_T = \left[\left\{\left(O_0 e^{(r+\kappa)\tau_1} - N_1^l S_{\tau_1}\text{LB}\right) e^{(r+\kappa)(\tau_2-\tau_1)} - N_2^l S_{\tau_2}\text{LB}\right\} e^{(r+\kappa)(\tau_3-\tau_2)} - N_3^l S_{\tau_3}\text{LB}\right] e^{(r+\kappa)(\tau_4-\tau_3)} \ldots$$

$O_{\tau_1^-}$ $O_{\tau_1^+}$ $O_{\tau_2^-}$

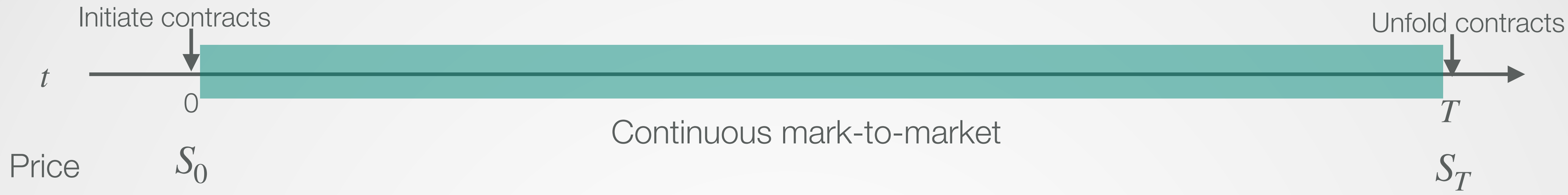
- Rearrange and get,

$$O_T = O_0 e^{(r+\kappa)T} - N_1^l S_{\tau_1}\text{LB}e^{(r+\kappa)(T-\tau_1)} - N_2^l S_{\tau_2}\text{LB}e^{(r+\kappa)(T-\tau_2)} - N_3^l S_{\tau_3}\text{LB}e^{(r+\kappa)(T-\tau_3)} - \ldots$$

- The collateral value is

$$C_T = (N_0 - N_1^l - N_2^l - N_3^l \ldots)S_T.$$

Chain up loans - Payoff



We write the payoff via indicator functions indicating how many liquidations are settled in $[0,T)$

$$
\begin{aligned}
(C_T - O_T)^+ = \quad & N_0(S_T - K_0 e^{(r+\kappa)T})^+ 1(\tau_1 > T) && \textbf{No liquidation} \\
& + N_1(S_T - K_1 e^{(r+\kappa)T})^+ 1(\tau_1 < T, \tau_2 > T) && \textbf{Exactly one liquidation} \\
& + N_2(S_T - K_2 e^{(r+\kappa)T})^+ 1(\tau_2 < T, \tau_3 > T) && \textbf{Exactly two liquidations} \\
& + \dots
\end{aligned}
$$

where $\tau_1 = \inf_{0 \le t \le T} \{t : \text{LTV}_t \ge \text{LTV}_H\}$, $\tau_i = \inf_{0 \le t \le T} \left\{t : \text{LTV}_t \ge \text{LTV}_H \text{ and } t > \tau_{i-1}\right\}$, and

$$
K_k = \frac{N_0}{N_k} K_0 = \frac{N_0}{N_k} \cdot \text{LTV}_0 S_0 \quad \forall k \in \mathbb{N} \text{ where } \frac{N_0}{N_k} = \left\{1 - \sum_{l=1}^{k} \left(\frac{\text{LTV}_H}{2\text{LB}}\right)^l\right\}^{-1}.
$$

Chain up loans - Price discontinuity

- Define an *overshooting parameter*

$$\xi_i \overset{\text{def}}{=} \frac{S_{\tau_i}}{H_i \cdot e^{(r+\kappa)\tau_i}}$$

- Then

$$\tau_i = \inf\left\{t : S_t \le H_i \cdot e^{(r+\kappa)t} \text{ and } t \ge \tau_{i-1}\right\}$$

$$H_i = \frac{H_1}{2^{i-1}} \prod_{j=1}^{i} \left(1 - \frac{\text{LTV}_H/2}{\xi_i \cdot \text{LB}}\right)^{-1}$$

- In addition

$$N_i = N_0 \prod_{j=1}^{i} \left(1 - \frac{\text{LTV}_H/2}{\xi_i \cdot \text{LB}}\right)^{-1}$$

$$K_i = \frac{K_0}{2^i} \prod_{j=1}^{i} \left(1 - \frac{\text{LTV}_H/2}{\xi_i \cdot \text{LB}}\right)^{-1}$$

Proof: Two period model payoff

When LTV_t hits LTV_H from below, AAVE allows liquidators to liquidate the borrower's position by half of the *debt*, i.e.

$$O_{\tau^+} = \frac{O_{\tau^-}}{2}.$$

We define the number of coins to be liquidated as $N_1^l = N_0 - N_1$, so we also have

$O_{\tau^+} = O_{\tau^-} - N_1^l S_{\tau^-}\text{LB}$. (LB = liquidation fee, discount)

Next, we write N_1^l in known terms,

$$\frac{O_{\tau^-}}{2} = N_1^l S_{\tau^-}\text{LB}$$

$$N_0 O_{\tau^-} = 2N_1^l N_0 S_{\tau^-}\text{LB}$$

$$N_0 O_{\tau^-} = 2N_1^l C_{\tau^-}\text{LB}$$

$$N_1^l = N_0 \frac{\text{LTV}_{\tau^-}}{2\text{LB}}$$

A useful expression ($\text{LTV}_{\tau^-} = \text{LTV}_H$ since price is continuous):

$$\frac{N_1}{N_0} = 1 - \frac{\text{LTV}_H}{2\text{LB}}$$

Therefore,

$$\begin{aligned}
C_{\tau^+} - O_{\tau^+} &= N_1 S_{\tau^+} - \frac{O_{\tau^-}}{2} e^{(r+\kappa)(\tau^+ - \tau^-)} \\
&= N_1 S_{\tau^+} - \frac{O_0 e^{r\tau^-}}{2} e^{(r+\kappa)(\tau^+ - \tau^-)} \\
&= N_1 S_{\tau^+} - \frac{C_0 \text{LTV}_0 e^{(r+\kappa)\tau^+}}{2} \\
&= N_1 S_{\tau^+} - \frac{N_0 S_0 \text{LTV}_0 e^{(r+\kappa)\tau^+}}{2} \\
&= N_1 \left(S_{\tau^+} - \frac{N_0}{N_1} \frac{1}{2} K_0 e^{(r+\kappa)\tau^+} \right) \\
&= N_1 \left(S_{\tau^+} - \frac{\text{LB}}{2\text{LB} - \text{LTV}_{\tau^-}} K_0 e^{(r+\kappa)\tau^+} \right)
\end{aligned}$$

If liquidator liquidates the position exactly with the price that breach the liquidation threshold, i.e. $\text{LTV}_{\tau^-} = \text{LTV}_H$, we have

$$(C_{\tau^+} - O_{\tau^+})^+ = N_1 \left(S_{\tau^+} - \frac{\text{LB}}{2\text{LB} - \text{LTV}_H} K_0 e^{(r+\kappa)\tau^+} \right)^+$$